13. RYZHOV O.S. and TERENT'YEV E.D., On the transition mode characterizing the triggering of a vibrator in a subsonic boundary layer on a plate. PMM, 50, 6, 1986.
14. ZHUK V.I. and RYZHOV O.S., On the stability of a freely interacting boundary layer. PMM, 45, 3, 1981.
15. TERENT'YEV E.D., The linear problem of a vibrator performing harmonic oscillations at supercritical frequencies in a subsonic boundary layer. PMM, 48, 2, 1984.
16. CRAIK A.D.D., The development of wave packets in unstable flows, Proc. Roy. Soc. Ser. A. 373, 1755, 1981.
17. RYZHOV O.S. and TERENT'YEV E.D., On a method of generating wave packets in a boundary layer. Modern Problems in the Mechanics of Continuous Media, Moscow, MFTI, 1985.
18. FEFORYUK M.V., The Saddle-point Method. Moscow, Nauka, 1977.
19. DRAZIN P.G. and REID W.H., Hydrodynamic stability. Cambridge Univ. Press 1981.
20. VASUDEVA B.R., Boundary-layer instability experiment with localized disturbance, J. Fluid Mech. 29, 4, 1967.

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THE PRESSURE ON A SPHERE WITH A DAMPING COATING WHEN A PLANE ACOUSTIC WAVE IS INCIDENT ON IT*

L.E. PEKUROVSKII, V.B. PORUCHIKOV and YU.A. SOZONENKO

In the problem on the interaction of an acoustic wave with a rigid sphere coated with a thin compressible layer /1/, the non-stationary pressure distribution on the sphere is found. The method of numerical inversion of the Laplace integral transform is used, together with asymptotic relations that hold in the case of a sufficiently thin coating. It is shown that the behaviour of the pressure is qualitatively different in the cases of a rigid sphere and a sphere with a coating. In the pressure-time dependence, successive series of oscillations are discovered, which are not seen with a rigid sphere, see /2, 3/. The pressure rise corresponding to the instant of interaction of the enveloping wave (the "Poisson spot" /4/) is displaced in time and in some cases exceeds twice the incident wave amplitude.

1. Formulation of the problem. Laplace transform of the pressure. At the instant $t=0$ let a plane acoustic wave of pressure p_i, previously propagating through a homogeneous fluid at rest with initial pressure p_0, density ρ_0, and sound velocity c_0, be incident on a rigid fixed sphere of radius a, coated with a thin damping layer of initial thickness h_0, where $h_0 \ll a$. The origin of a system of spherical coordinates r, ψ, φ is at the centre of the sphere, the incident wave front is perpendicular to the z axis z $(z = r\cos\varphi)$, and the motion is in the negative direction of the axis. For simplicity, the incident wave is regarded as a step with a pressure drop p_m

$$p_i = p_m \eta\left(t + \frac{z-a}{c_0}\right) + p_0, \quad \eta(t) = \begin{cases} 0, t \leqslant 0 \\ 1, t > 0 \end{cases} \tag{1.1}$$

We introduce the dimensionless pressure disturbances $\bar{p}$ and $\bar{p}_i$, the time $\bar{t}$, and coordinate $\bar{r}$ in accordance with the relations

$$\bar{p} = \frac{p-p_0}{p_m}, \quad \bar{p}_i = \frac{p_i - p_0}{p_m}, \quad \bar{t} = \frac{c_0 t}{a}, \quad \bar{r} = \frac{r}{a}$$

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where p, p_i, t, r are the corresponding dimensional quantities.

Below, we omit the bar over dimensionless quantities.

For the disturbance p_s introduced by the sphere into the incident wave pressure field we have the relations /1/

$$p_s = p - p_i, \quad p_i = \eta\,(r\cos\varphi - 1 + t) \tag{1.2}$$

$$\frac{\partial^2 p_s}{\partial r^2} + \frac{2}{r}\frac{\partial p_s}{\partial r} + \frac{1}{r^2}\frac{\partial^2 p_s}{\partial \varphi^2} + \frac{\operatorname{ctg}\varphi}{r^2}\frac{\partial p_s}{\partial \varphi} = \frac{\partial^2 p_s}{\partial t^2}, \quad t > 0, \quad r > 1$$

$$p_s = 0, \quad t \leqslant 0$$

$$\frac{\partial^2 (p_s + p_i)}{\partial t^2} = \frac{1}{\gamma}\frac{\partial (p_s + p_i)}{\partial r}, \quad r = 1 \quad \left(\gamma = \frac{\rho_0 c_0^2}{\rho_* c_*^2}\frac{h_0}{a}\right)$$

The last boundary condition in (1.2), which models the presence of the damping coating, corresponds to the assumption of linear dependence of the layer thickness on the pressure, which is regarded as constant across the layer. Since th problem is linear, it can be referred to a sphere with $r = 1$. In additon, we neglect for simplicity the possibility of flows in the layer along the body surface (it was shown in /1/ that this can be done provided that the sound velocity in the fluid is much greater than the sound velocity in the compressible layer). The parameter γ characterizes the softness of the layer (ρ_*, c_* are the density and sound velocity in the layer), in fact: if $\gamma \to \infty$, the last boundary condition in (1.2) becomes $p =$ const (the condition on an absolutely soft body with given pressure at the boundary), while if $\gamma \to 0$, it becomes the condition $\partial p / \partial r = 0$ on an absolutely rigid sphere.

Applying the Laplace integral transformation with respect to time in system (1.2) and solving the resulting boundary value problem, we find

$$p^*(s, r, \varphi) = \int_{-\infty}^{\infty} p\,(t, r, \varphi)\, e^{-st}\, dt =$$

$$\frac{e^{-s}}{s}\sqrt{\frac{2\pi}{sr}} \sum_{n=0}^{\infty}\left(n + \frac{1}{2}\right) C_n(s, r)\, P_n(\cos\varphi)$$

$$C_n(s, r) = I_{n+1/2}(sr) - \frac{2sI'_{n+1/2}(s) - (1 + 2\gamma s^2)\, I_{n+1/2}(s)}{2sK'_{n+1/2}(s) - (1 + 2\gamma s^2)\, K_{n+1/2}(s)} K_{n+1/2}(sr)$$

($\operatorname{Re} s > 0$, since $p_i = p_s \equiv 0$ for $t < 1 - r\cos\varphi$).

Here, $I_{n+1/2}(s)$, $K_{n+1/2}(s)$ are the modified Bessel functions of the first and third kinds respectively, and $P_n(x)$ is a Legendre polynomial.

Putting $r = 1$, we find the boundary value of p^* on the sphere /1/

$$p^*(s, 1, \varphi) = \frac{e^{-s}}{s}\sqrt{\frac{2\pi}{s}} \sum_{n=0}^{\infty} \frac{(2n+1)\, P_n(\cos\varphi)}{(1 + 2\gamma s^2)\, K_{n+1/2}(s) - 2sK'_{n+1/2}(s)} \tag{1.3}$$

Using well-known relations for the Bessel functions /5/, we can write (1.3) in the shorter form

$$p^* = \frac{1}{s}\sum_{n=0}^{\infty} \frac{(2n+1)\, P_n(\cos\varphi)}{(\gamma s^2 - n)\, R\,(n + 1/2, s) + sR\,(n + 3/2, s)} \tag{1.4}$$

$$R\left(n + \frac{1}{2}, s\right) = \sum_{k=0}^{\infty} \frac{(n+k)!}{k!\,(n-k)!}\frac{1}{(2s)^k}$$

It is not possible to obtain the exact form of the original from the image (1.4), so that a method of numerical inversion of the transform will be given later in Para.3. In the meantime we consider some approximate methods whereby fairly simple relations, suitable for qualitative analysis, can be obtained for the pressure on the sphere.

2. Approximate relations for the pressure. We consider the asymptotic expansion of the solution (1.3) for an extremely soft coating as $\gamma \to \infty$. We write the original of (1.3) in the form

$$p\,(t, 1, \varphi) = \sum_{n=0}^{\infty} T_n(t)\, P_n(\cos\varphi) \tag{2.1}$$

$$T_n(t) = \frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty} T_n^*(s)\, e^{st}\, ds, \quad T_n^*(s) = \frac{(n + 1/2)\sqrt{2\pi}}{e^s A_n(s)} \quad (b > 0) \tag{2.2}$$

$$A_n(s) = \gamma s^3 \sqrt{s} K_{n+1/2}(s) \varkappa(s), \quad \varkappa(s) = 1 - \frac{(K_{n+1/2}(s)/\sqrt{s})'}{\gamma K_{n+1/2}(s)\sqrt{s}} \tag{2.3}$$

Using the properties of the function $K_{n+1/2}(s)$, we can show that

1) the integrand in (2.2) is an analytic function in the domain $\operatorname{Re} s > 0$, while in the remainder of the s plane it has no singularities other than poles;

2) the function $T_0^*(s)$ has three poles, one at the point $s = 0$ and the other two in the domain $\operatorname{Re} s < 0$; the latter tend to $s = 0$ like $1/\sqrt{\gamma}$ as $\gamma \to \infty$;

3) for $n \geqslant 1$, $T_n^*(s)$ has $n + 2$ poles, all located in the domain $\operatorname{Re} s < 0$;

4) depending on the value of γ $(0 \leqslant \gamma < \infty)$, the poles of $T_n^*(s)$ $(n \geqslant 0)$ may be either real or complex (in the latter case they come in conjugate pairs);

5) n poles of $T_n^*(s)$, as $\gamma \to \infty$, tend to zeros of $K_{n+1/2}(s)$ $(n \geqslant 1)$, while the other two tend to the value $s = 0$ like $\sqrt{(n+1)/\gamma}$.

Since the zeros of $K_{n+1/2}(s)$ $(n \geqslant 1)$ are at a finite distance from the imaginary axis (it can be shown that the distance is lower bounded uniformly with respect to n, i.e., $\operatorname{Re} s_m \leqslant -1$, where s_m runs over the zeros of $K_{n+1/2}(s), (n \geqslant 1)$), their contribution to the pressure component $T_n(t)$ is of the order of $e^{-\alpha t}$, $\alpha \geqslant 1$, provided that t is not too close to zero $(t \geqslant t_0)$. Consequently, if the contour of integration with respect to s in (2.2) is deformed into the left half-plane, the main contribution to $T_n(t)$ for large γ and $t \geqslant t_0$ will come from the residues at the remaining two poles, located near the point $s = 0$, and also from the residue at $s = 0$ when $n = 0$.

In the case when $n = 0$ the three poles of $A_n(s)$ mentioned in Para.2 have the form, when γ is large,

$$s_{1,2}^{(0)} = -\frac{1}{2\gamma} \pm \frac{i}{\sqrt{\gamma}}\left(1 - \frac{1}{8\gamma}\right) + O(\gamma^{-5/2}) \qquad s_3^{(0)} = 0$$

so that

$$T_0(t) = 1 - \exp\left(-\frac{t}{2\gamma}\right)\left[\cos\frac{t}{\sqrt{\gamma}} + \frac{1}{2\sqrt{\gamma}}\sin\frac{t}{\sqrt{\gamma}} + O\left(\frac{1}{\gamma^{3/2}}\right)\right] \tag{2.4}$$

where the estimate is uniform with respect to t $(0 \leqslant t < \infty)$.

If $n \geqslant 1$, then, since $\sqrt{s} K_{n+1/2}(s)$ has no zeros in the neighbourhood of $s = 0$, the pair of complex conjugate roots of $A_n(s)$ in the neighbourhood of $s = 0$ is given as $\gamma \to \infty$ by the equation

$$\varkappa(s) = 0 \tag{2.5}$$

To solve this equation, we use the asymptotic form as $s \to 0$ of the cylindrical functions /5/:

$$K_\nu(s) = \frac{\Gamma(\nu)}{2}\left(\frac{2}{s}\right)^\nu \left[1 + \left(\frac{s}{2}\right)^2 \frac{1}{1-\nu}\right] + O(s^{4-\nu}) + O(s^\nu), \ \nu > 1 \tag{2.6}$$

Substituting (2.6) into (2.5), we can find the expressions for the required roots of $A_n(s)$ with $n \geqslant 1$ and $\gamma \to \infty$:

$$s_{1,2}^{(n)} = \frac{-\pi(n+1)^n}{2^{2n+1}\Gamma^2(n+1/2)\gamma^{n+1}}\left[1 + O\left(\frac{1}{\gamma}\right)\right] \pm \sqrt{\frac{n+1}{\gamma}}\, i\left[1 - \frac{1}{2\gamma(2n-1)} + O\left(\frac{1}{\gamma^2}\right)\right] \tag{2.7}$$

As remarked above, the remaining n poles tend, as $\gamma \to \infty$, to the zeros of $K_{n+1/2}(s)$. As a result, using (2.2), we find by the theorem of residues

$$T_1(t) = \frac{3}{\gamma}e^{-t} + 3\exp\left(-\frac{t}{\gamma^2}\right)\left[\frac{1}{\sqrt{2\gamma}}\sin\sqrt{\frac{2}{\gamma}}\,t - \frac{1}{\gamma}\cos\sqrt{\frac{2}{\gamma}}\,t + O\left(\frac{1}{\gamma^{3/2}}\right)\right] \tag{2.8}$$

$$T_2(t) = -\frac{5}{3\gamma}\exp\left(-\frac{3}{2}t\right)\left(\cos\frac{\sqrt{3}}{2}t + \sqrt{3}\sin\frac{\sqrt{3}}{2}t\right) + \frac{5}{3\gamma}\exp\left(-\frac{t}{2\gamma^3}\right)\left[\cos\sqrt{\frac{3}{\gamma}}\,t + O\left(\frac{1}{\gamma^{3/2}}\right)\right]$$

$$T_n(t) = \exp\left[-\frac{\pi(n+1)^n}{2^{2n+2}\Gamma^2(n+1/2)}\frac{t}{\gamma^{n+1}}\right] O\left(\frac{1}{\gamma^{n/2}}\right) + O\left(\frac{e^{-\alpha t}}{\gamma}\right) \tag{2.9}$$

$$n \geqslant 3, \quad \alpha > 3/2$$

In (2.8) the residues are included from the zeros of $K_{3/2}(s)$ and $K_{5/2}(s)$ respectively, in order to satisfy the relations $T_1(0) = T_2(0) = 0$. The estimate in $T_1(t)$ and $T_2(t)$ is satisfied uniformly with respect to t for $0 \leqslant t < \infty$.

In short, we obtain the following asymptotic expression, suitable for $\gamma \gg 1$ and $t \geqslant t_0$, for the pressure on the sphere:

$$p(t,\ 1,\ \varphi) = T_0(t) + T_1(t)\cos\varphi + T_2(t)(3\cos^2\varphi - 1)/2 + O(\gamma^{-1/2}) + O(e^{-\alpha t}\gamma^{-1}),\ \alpha > 3/2 \tag{2.10}$$

where $T_n(t)\ (n = 0, 1, 2)$ are given by (2.4), (2.8).

In addition to (2.10) we can propose another approximate relation /1/, which is obtained by piston theory and describes the pressure behaviour in the head part of the sphere during a time interval from the instant when incident wave diffraction starts:

$$p(t, 1, \varphi) = \left[1 - \exp(-t_1)\left(\operatorname{ch}\beta t_1 - \frac{1 + 2\cos\varphi}{\beta}\operatorname{sh}\beta t_1\right)\right]\eta(t_1) \tag{2.11}$$

$$t_1 = \frac{t - 1 + \cos\varphi}{2\gamma}, \qquad \beta = \begin{cases} \sqrt{1 - 4\gamma}, & \gamma \leqslant 1/4 \\ i\sqrt{4\gamma - 1}, & \gamma > 1/4 \end{cases}$$

3. **Numerical inversion of the Laplace transform.** Our method is based on direct evaluation of the inversion integral

$$p(t, 1, \varphi) = \frac{1}{2\pi i}\int\limits_{\xi_0 - i\infty}^{\xi_0 + i\infty} p^*(s, 1, \varphi)\, e^{st}\, ds, \quad \xi_0 > 0 \tag{3.1}$$

where $p^*(s, 1, \varphi)$ is given by (1.4).

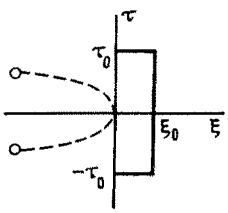

Fig.1

All the poles of $p^*(s, 1, \varphi)$ are in the left half-plane $\xi < 0$ of the s plane (except for the one pole at $s = 0$); some of them, being complex conjugate, approach the $\xi = 0$ axis as the parameter γ increases, along trajectories shown qualitatively by the broken line in Fig.1. In order for the computational method to be suitable for any value of γ, the contour of integration is chosen in the form of a step line (continuous in Fig.1):

$$s = \begin{cases} \xi_0 + i\tau, & -\tau_0 \leqslant \tau \leqslant \tau_0 \\ \xi \pm i\tau_0, & 0 \leqslant \xi \leqslant \xi_0 \\ \pm i\tau, & -\tau_0 \leqslant \tau,\ \tau \geqslant \tau_0 \end{cases} \tag{3.2}$$

The parameters ξ_0 and τ_0 are aribtrary, and can be chosen so as to shorten the time and increase the accuracy.

Using the form (3.2) of the contour of integration, we can rewrite (3.1) as

$$p(t, 1, \varphi) = I_1 - I_2 + I_3$$

$$I_1 = \frac{e^{\xi_0 t}}{\pi}\int\limits_0^{\tau_0} \operatorname{Re}\left[p^*(\xi_0 + i\tau, 1, \varphi)\, e^{it\tau}\right] d\tau$$

$$I_2 = \frac{1}{\pi}\int\limits_0^{\xi_0} |\, p^*(\xi + i\tau_0, 1, \varphi)\,|\, e^{\xi t}\sin\left[\theta(\xi + i\tau_0) + t\tau_0\right] d\xi$$

$$I_3 = \frac{1}{\pi}\int\limits_{\tau_0}^{\infty} \operatorname{Re}\left[p^*(i\tau, 1, \varphi)\, e^{it\tau}\right] d\tau$$

where $\theta(\xi + i\tau)$ is the argument of $p^*(\xi + i\tau, 1, \varphi)$. Here we use the fact that p^* is complex selfconjugate as a function of s, as follows from (1.4).

The integrands in I_1 and I_3 are sums of series, and may oscillate and be slow to evaluate. We therefore devised a special adaptive subroutine for computing them, based on Filon's quadrature formula /5/ (adaptive in the sense of automatically determined the size of the integrands in such a way that the result satisfies a preassigned accuracy). The subroutine is similar to the QUANC 8 subroutine /6/ for evaluating integrals using the Newton-Cotes quadrature formula, in which the interpolation polynomial has degree eight. We used the QUANC 8 subroutine directly to evaluate I_2.

4. **The pressure on the sphere.** The time dependence of the pressure in the head part

of the coated sphere is shown by the continuous curves of Fig.2 for different values of γ. For a rigid sphere, when $\gamma = 0$, the pressure initially falls to the value $p = 2$; whereas when $\gamma \neq 0$, a similar fall is seen only after the pressure reaches a local maximum, before which it is increasing from zero. It is interesting that there are marked oscillations of the pressure in sufficiently long time intervals, the maxima in these oscillations being usually greater than the initial maximum in the initial period.

The time dependence of the pressure in the head part of an absolutely rigid sphere ($\gamma = 0$) was earier obtained by a finite-difference approximation of the wave equation; it is shown in Fig.30 of /7/, but only up to the instant $t = 5$, when it is still well below its limiting value of unity as $t \to \infty$. It can be seen from Fig.2 that this value is established in practice, and then quite sharply, soon after the instant $t = 1 + 3\pi/2$, when the diffracted wave front returns to the head part after travelling round the sphere.

The curves relevant to the approximate expression (2.11) are shown broken in Fig.2. Comparing them with the continuous curves, we see that the piston theory on which (2.11) is based is reasonably accurate for the initial time interval and continues for some time afterwards to give qualitatively correct results, though it cannot predict the serious pressure oscillations that then occur. It must be said, however, that the fact of the oscillations follows from (2.11), though the exponential factor in (2.11) leads to a monotonic decrease of the oscillation amplitude.

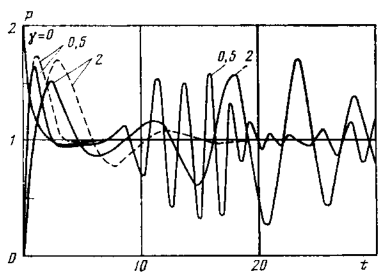

Fig.2

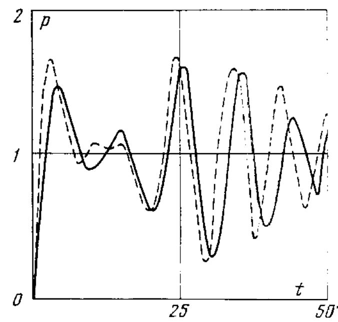

Fig.3

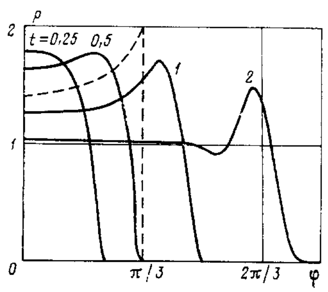

Fig.4

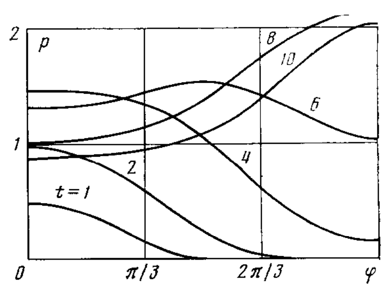

Fig.5

The calculations by numerical integration (continuous curves) are compared with the calculations from asymptotic expression (2.10) (the broken curves) in Fig.3 for the case $\gamma = 5$ ($\varphi = 0$). Even with this fairly small value of γ the asymptotic expression describes the time dependence of the pressure quite well, while with $\gamma = 50$ it gives virtually the same results as numerical integration (we omit the comparison for brevity, see /8/).

Three terms in (2.10) enable three stages in the time dependence of the pressure to be distinguished: the first, lasting for a time of the order of 2γ, during which there are oscillations with amplitude of order $1/(2\sqrt{\gamma})$ and with frequency $\sim\sqrt{1/\gamma}$, the second, which lasts of order γ^2, during which there are oscillations of amplitude of order $3/\sqrt{2\gamma}$ and frequency $\sim\sqrt{2/\gamma}$, and the third, which is the longest, of order $2\gamma^3$, but with oscillations of lower amplitude, of order $5/(3\gamma)$ and higher frequency of order $\sqrt{3/\gamma}$. The stages can be continued further, and if they are assigned the number n, their duration increases as γ^n, while the amplitude of oscillation decreases as $\gamma^{-(n-1)/2}$ and their frequency increases as $\sqrt{n/\gamma}$. These features of the pressure time dependence are confirmed by calculations, notably as shown in Fig.3.

The evolution of the pressure distribution on the sphere is shown as a function of the angle φ in Fig.4 for $\gamma = 0, 1$, and different instants t. For comparison, we also show the curve (broken) for a rigid sphere ($\gamma = 0$), for the instant $t = 0, 5$. The pressure distributions close to the diffracted wave front are clearly qualitatively different.

In Fig.5 we show curves of the pressure distribution on a sphere with a softer damping coating ($\gamma = 5$) for different instants t. The pressure rise at the point $\varphi = \pi$, due to interaction of waves travelling round the sphere, can prove to be substantial and in some cases is more than twice the incident-wave amplitude.

REFERENCES

1. PEKUROVSKII L.E., PORUCHIKOV V.B. and SOZONENKO YU.A., Interaction of an acoustic wave with a body coated by a thin compressible layer, PMM, 47, 5, 1983.
2. GUZ A.N. and KUBENKO V.D., Theory of non-stationary aero-elastic shells, Methods of claculating shells, 5, Nauk. Dumka, Kiev, 1982.
3. GRIZOLYUK E.I. and GORSHKOV A.G., Non-stationary hydro-elasticity of shells, Sudostroenie, Leningrad, 1974.
4. HÖNL H., MAUE A.W.and WESTPFAHL K., Theorie der Beugung, Handbuch der Physik, 25/1, Springer, Berlin, 1961.
5. ABRAMOWITZ M. and STIGUN A., (Editors), Handbook of mathematical functions with formulas graphs, and mathematical tables, Nat. Bureau of Standards, Washington, 1964.
6. FORSYTHE G.E., MALCOLM M.A. and MOLER C.B., Computer methods for mathematical computations, Englewood Cliffs, 1977.
7. SLEPYAN L.I., Non-stationary elastic waves (Nestatsionarnye uprugie volny), Sudostroenie, Moscow, 1972.
8. SOZONENKO YU.A. (Ed.), Diffraction of acoustic waves by a body with a damping coating, Izd-vo MGU, Moscow, 1985.

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STATIONARY VIBRATIONS OF AN ELASTIC HALF-SPACE WITH A CIRCULAR CYLINDRICAL CAVITY SUBJECTED TO A PERIODIC LOAD*

L.A. ALEKSEYEVA

The problem of the stationary vibrations of an elastic half-space with a circular cylindrical cavity subjected to a periodic load along the axis is considered to investigate the state of stress and strain of extended shallow mining shafts under dynamic effects. The problem is reduced to the solution of a system of equations with normal-type determinant by the method of superposition of solutions by using contour integrals of Fourier type and Fourier-Bessel series. The question of the existence and uniqueness of the solution is examined, and its singularities are investigated as a function of the velocity of the moving load or its period. It is shown that Rayleigh surface waves occur in the medium for velocities above the Rayleigh value.

1. Formulation of the problem.

Let us consider an isotropic elastic half-space $x \leqslant h, h > 0$ with Lamé parameters λ, μ, ρ, weakened by a circular cylindrical cavity of radius $R, R < h$ (Fig.1), whose axis OZ is parallel to the half-space boundary. We connect a cylindrical coordinate system (O, r, θ, z) to the cylinder axis, whose polar axis coincides with the OX axis. A load that is stationary in t and periodic in z acts on the cylinder cavity

$$\sigma_{rj} = 2\mu\varepsilon_j p_j(\theta)\, e^{i(\xi z-\omega t)} \tag{1.1}$$

$$j = r, \theta, z;\ \varepsilon_r = 1,\ \varepsilon_\theta = \varepsilon_z = i$$

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